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Vector condensate model of electroweak interactions

G. Cynolter[1,a], E. Lendvai[1], G. Pócsik[2]

[1] Theoretical Physics Research Group of Hungarian Academy of Sciences, Eötvös University, Budapest, Hungary
[2] Institute for Theoretical Physics, Eötvös Loránd University, Budapest, Hungary

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Abstract. In the standard model of the electroweak interactions the Higgs doublet is replaced by a complex vector doublet and a real vector singlet. The gauge symmetry is broken dynamically by a mixed condensate of the doublet and singlet vector fields. Gauge fields get their usual standard model masses by condensation. The new vector matter fields become massive by their gauge invariant self-couplings. The model cannot be renormalized perturbatively. Fermions are assigned to the gauge group in the usual manner. Fermion masses are coming from a gauge invariant fermion–vector field interaction by a mixed condensate. The Kobayashi–Maskawa description is unchanged. It is shown that from the new matter fields a large number of spin-one particle pairs is expected at future high energy e^+e^- linear colliders of 500–1500 GeV.

A current description of electroweak symmetry breaking is through a weakly interacting scalar doublet. Another possibility is a symmetry breaking system interacting strongly with the longitudinal weak vector bosons which has been realised in the DHT model [1] based on a chiral Lagrangian approach. An alternative description of the strongly interacting symmetry breaking system has been proposed in the BESS model [2] through non-linear realizations. Top quark condensation has also been suggested for describing the electroweak symmetry breaking [3] leading to several interesting studies e.g. [4]. Electroweak symmetry breaking caused by the condensation of a vector field was studied too [5]. Condensation of vector bosons in different scenarios was considered in the literature [6]. Recently, little Higgs models [7] attracted attention.

In the present note we start with the usual Lagrangian of the standard model of electroweak interactions, but instead of the scalar doublet two new matter fields are introduced. One of them is a $Y = 1$, $T = 1/2$ doublet of complex vector fields:

$$B_\mu = \begin{pmatrix} B_\mu^{(+)} \\ B_\mu^{(0)} \end{pmatrix}, \tag{1}$$

the other is a real $Y = 0$, $T = 0$ vector field C_μ. This extends our recent model [5] where only B_μ was present with the condensation of $B_\mu^{(0)}$. Consequently, we are able to describe a more complete symmetry breaking and to generate fermion masses from a gauge invariant interaction Lagrangian while the mass ratio of $B^{(+)}$ and $B^{(0)}$ does not become fixed. The key point is the introduction of a mixed B_μ–C_μ condensate together with suitable gauge invariant interactions of the new matter fields. This leads to non-vanishing standard model particle masses, as well as B, C particle masses. It turns out that altogether three condensates emerge, but only one combination of them is fixed by the Fermi coupling constant. The model should be considered as a non-renormalizable low energy effective one, having a cutoff scale of a few TeV. Its new particle content is a charged vector boson pair and three neutral vector bosons. As is shown, these can be pair produced in e^+e^- annihilation, and at future linear colliders of 500–1500 GeV they can provide a large number of events.

To build the model, in the Lagrangian of the standard model the interactions of the scalar doublet are replaced by the gauge invariant Lagrangian

$$\begin{aligned} L_{BC} = &-\frac{1}{2}\overline{(D_\mu B_\nu - D_\nu B_\mu)}\,(D^\mu B^\nu - D^\nu B^\mu) \\ &-\frac{1}{2}\,(\partial_\mu C_\nu - \partial_\nu C_\mu)\,(\partial^\mu C^\nu - \partial^\nu C^\mu) - V(B,C)\,, \end{aligned} \tag{2}$$

where D_μ is the covariant derivative, $g_{\mu\nu} = + - - -$, and for the potential $V(B,C)$ we assume

$$V(B,C) = \lambda_1\left(\overline{B}_\nu B^\nu\right)^2 + \lambda_2\left(C_\nu C^\nu\right)^2 + \lambda_3 \overline{B}_\nu B^\nu C_\mu C^\mu\,, \tag{3}$$

depending only on the B-, C- lengths. Other quartic terms would not change the argument. The $\lambda_{1,2,3}$ are real and from positivity

$$\lambda_1 > 0\,, \quad \lambda_2 > 0, \quad 4\lambda_1\lambda_2 > \lambda_3^2\,. \tag{4}$$

Mass terms to be generated are not introduced explicitly in (3). Fermion–BC interactions are introduced later on.

To break the gauge symmetry, we assume a non-vanishing mixed condensate in the vacuum,

$$\left\langle \overline{B}_\mu C_\nu \right\rangle = g_{\mu\nu}\ (0, d)\,, \tag{5}$$

[a] e-mail: cyn@general.elte.hu

where the left-hand side could be rotated into $(0,d)$, $d \neq 0$, respecting also electric charge conservation and defining the neutral and charged components in (1). By $U_Y(1)$ d can be chosen real. A real d respects also combined TCP and C symmetries. By TCP-invariance (5) equals $\langle C_\nu \overline{B}_\mu \rangle$. It follows from (5) that the only non-vanishing mixed condensate is

$$\langle B_{1\mu} C_\nu \rangle = \sqrt{2} g_{\mu\nu} d \,, \tag{6}$$

with

$$B_\mu^{(0)} = \frac{1}{\sqrt{2}} \left(B_{1\mu} + \mathrm{i} B_{2\mu} \right) \,, \tag{7}$$

where $B_{j\mu}$ is real. Once there exists the mixed condensate, B and C are assumed to condense separately:

$$\begin{aligned} \langle \overline{B}_\mu B_\nu \rangle &= g_{\mu\nu} k_1 \,, \quad k_1 \neq 0 \,, \\ \langle C_\mu C_\nu \rangle &= g_{\mu\nu} k_3 \,, \quad k_3 \neq 0 \,. \end{aligned} \tag{8}$$

In more detail, assume that k_1 originates from $B_\mu^{(0)}$ condensation,

$$\begin{aligned} \left\langle B_\mu^{(+)\dagger} B_\nu^{(+)} \right\rangle &= 0 \,, \\ \left\langle B_\mu^{(0)\dagger} B_\nu^{(0)} \right\rangle &= g_{\mu\nu} k_1 \,. \end{aligned} \tag{9}$$

Equation (9) reproduces the pattern of gauge particle masses [5]. All the condensates linear in $B_\mu^{(+)}$ vanish by charge conservation. Finally, we assume in advance that

$$\left\langle B_\mu^{(0)} B_\nu^{(0)} \right\rangle = \left\langle B_\mu^{(0)\dagger} B_\nu^{(0)\dagger} \right\rangle = g_{\mu\nu} \ k_2 \,. \tag{10}$$

The point is that in general $B_{1\mu}$ and $B_{2\mu}$ belong to different masses, so that $k_2 \neq 0$. $k_{1,2,3}$ are real and $k_1 < 0$, $k_3 < 0$, as shown by particle masses and simple models. The condensates are of non-perturbative origin caused by the strong interaction $V(B,C)$. Among them only k_1 is fixed by contemporary phenomenology.

Mass terms are obtained in the linearized form of L_{BC} via condensates. The $W^\pm$ mass is determined by the total B-condensate, while the two neutral gauge field combinations are proportional to $B_\mu^{(+)\dagger} B_\nu^{(+)}$ and $B_\mu^{(0)\dagger} B_\nu^{(0)}$, respectively. Therefore, the assumption (9) yields

$$m_{\text{photon}} = 0, \ m_W = \frac{g}{2}\sqrt{-6k_1}, \ m_Z = \frac{g}{2\cos\theta_{\mathrm{W}}} \sqrt{-6k_1} \,. \tag{11}$$

Low energy phenomenology gives

$$k_1 = -\left(6\sqrt{2} G_{\mathrm{F}}\right)^{-1}, \ (-6k_1)^{1/2} = 246\,\mathrm{GeV} \,. \tag{12}$$

$B^\pm$ and B_2 get the following masses:

$$m_\pm^2 = -8\lambda_1 k_1 - 4\lambda_3 k_3 \,, \tag{13}$$

$$m_{B_2}^2 = -10\lambda_1 k_1 + 2\lambda_1 k_2 - 4\lambda_3 k_3 = m_\pm^2 + 2\lambda_1 (k_2 - k_1) \,.$$

For $\lambda_3, -k_1, -k_3 > 0$, $\ m_{B_2}^2 > m_\pm^2 > 0$ since $k_2 > k_1$. The B_1–C sector is slightly more complicated; here one arrives at the following bilinear combinations in the potential for $B_{1\mu}, C_\mu$:

$$V(B,C) \to -\frac{m_1^2}{2} B_{1\mu} B^{1\mu} - \frac{m_2^2}{2} C_\nu C^\nu - m_3^2 B_{1\mu} C^\mu \,, \tag{14}$$

with

$$\begin{aligned} -m_1^2 &= 10\lambda_1 k_1 + 2\lambda_1 k_2 + 4\lambda_3 k_3 = -m_{B_2}^2 + 4\lambda_1 k_2 \,, \\ -m_2^2 &= 24\lambda_2 k_3 + 8\lambda_3 k_1 \,, \\ -m_3^2 &= 4\sqrt{2}\lambda_3 d \,. \end{aligned} \tag{15}$$

Here $m_1^2 > 0$ is $k_1 + k_2 < 0$; $m_2^2 > 0$ and $m_3^2 \lesseqgtr 0$. A positive potential in (14) requires

$$m_1^2, m_2^2 > 0 \,, \quad m_1^2 m_2^2 > m_3^4 \,. \tag{16}$$

Equation (14) shows that $B_{1\mu}$ and C_μ are non-physical fields; the mass eigenstates are defined by

$$\begin{aligned} B_{f\mu} &= c B_{1\mu} + s C_\mu \,, \\ C_{f\mu} &= -s B_{1\mu} + c C_\mu \,, \end{aligned} \tag{17}$$

where $c = \cos\phi$, $\ s = \sin\phi$, and ϕ denotes the mixing angle defined by

$$\frac{1}{2} \sin 2\phi (m_1^2 - m_2^2) = \cos 2\phi m_3^2 \,. \tag{18}$$

The physical masses are

$$\begin{aligned} m_{B_f}^2 &= c^2 m_1^2 + s^2 m_2^2 + 2cs m_3^2 \,, \\ m_{C_f}^2 &= s^2 m_1^1 + c^2 m_2^2 - 2cs m_3^2 \,, \end{aligned} \tag{19}$$

whence

$$2m_{B_f, C_f}^2 = m_1^2 + m_2^2 \pm \frac{m_1^2 - m_2^2}{\cos 2\phi} \,. \tag{20}$$

For $(m_1^2 - m_2^2)/\cos 2\phi > 0$ (< 0) $m_{B_f}^2 > m_{C_f}^2 > 0$ $(m_{C_f}^2 > m_{B_f}^2 > 0)$. At vanishing mixing, $m_3^2 = 0$, $B_{1\mu}$ and C_μ become independent having the masses m_1 and m_2; taking $k_2 = 0$ and omitting C_μ we recover the model of [5]. k_2 shifts the real component field masses from the mass of the imaginary part $B_{2\mu}$.

The particle spectrum of the B–C sector consists of the spin-one $B^\pm$ and the three neutral spin-one particles B_2, B_f, C_f. Their masses are rather weakly restricted. Beside the gauge coupling constants and $\lambda_1, \lambda_2, \lambda_3$, the model has three basic condensates $\langle V_{i\mu} V_{i\nu} \rangle$, $V_{i\mu} = B_{2\mu}, B_{f\mu}, C_{f\mu}$. The k_1, k_2, k_3, d condensates are built up from these as follows:

$$g_{\mu\nu} d = \frac{1}{\sqrt{2}} cs \left(\langle B_{f\mu} B_{f\nu} \rangle - \langle C_{f\mu} C_{f\nu} \rangle \right) \,,$$

$$g_{\mu\nu} k_1 = \frac{1}{2} \left\{ c^2 \langle B_{f\mu} B_{f\nu} \rangle + s^2 \langle C_{f\mu} C_{f\nu} \rangle + \langle B_{2\mu} B_{2\nu} \rangle \ \right\} \,,$$

$$g_{\mu\nu} k_2 = \frac{1}{2} \left\{ c^2 \langle B_{f\mu} B_{f\nu} \rangle + s^2 \langle C_{f\mu} C_{f\nu} \rangle - \langle B_{2\mu} B_{2\nu} \rangle \ \right\} \,,$$

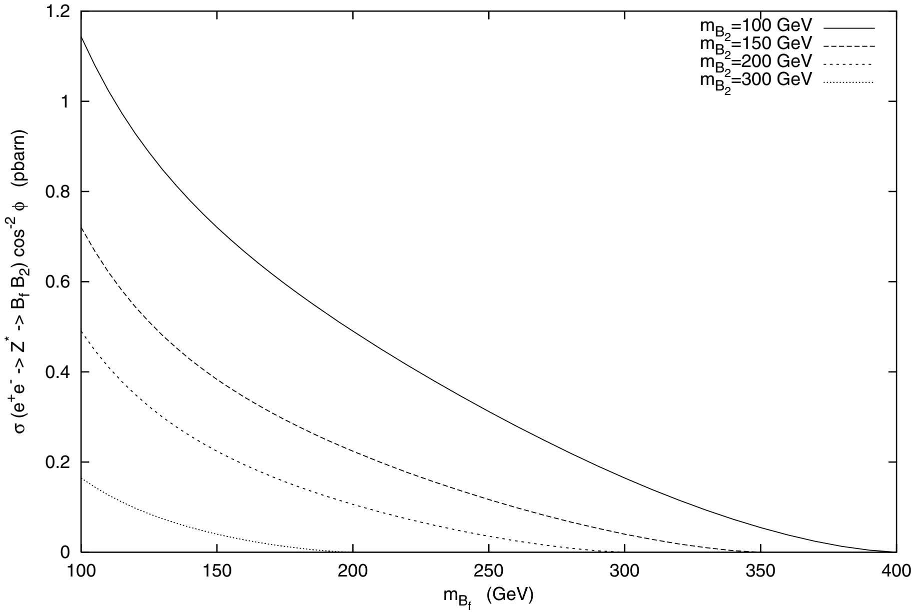

Fig. 1. $\cos^{-2}\phi\,\sigma\left(e^+e^- \to B_f B_2\right)$ versus m_{B_f} at $\sqrt{s} = 500\,\mathrm{GeV}$ and various m_{B_2}

$$g_{\mu\nu}k_3 = s^2\left\langle B_{f\mu}B_{f\nu}\right\rangle + c^2\left\langle C_{f\mu}C_{f\nu}\right\rangle\,. \tag{21}$$

From (21) d can be written as

$$2\sqrt{2}\cot 2\phi\, d = k_1 + k_2 - k_3\,. \tag{22}$$

Turning to the dynamical fermion mass generation, we add to the gauge vector and matter vector field Lagrangians the usual fermion–gauge vector Lagrangian, as well as a new gauge invariant piece responsible for the fermion matter–vector field interactions and in the usual notation this is (for quarks)

$$g^u_{ij}\overline{\psi}_{i\mathrm{L}}u_{j\mathrm{R}}B^C_\nu C^\nu + g^d_{ij}\overline{\psi}_{i\mathrm{L}}d_{j\mathrm{R}}B_\nu c^\nu + \mathrm{h.c.}\,, \tag{23}$$

$$\psi_{i\mathrm{L}} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad B^C_\nu = \begin{pmatrix} B^{(0)\dagger}_\nu \\ -B^{(+)\dagger}_\nu \end{pmatrix}.$$

Clearly the mixed condensate provides fermion masses and also the Kobayashi–Maskawa description is unchanged. A typical fermion mass is

$$m_f = -4g_f d\,, \tag{24}$$

and only $g_f d$ becomes fixed but $m_{f1}/m_{f2} = g_{f1}/g_{f2}$ as usual. If d is about $k_1 \simeq G_\mathrm{F}^{-1}$, then g_f is a factor of $G_\mathrm{F}^{1/2}$ weaker than the approximate standard model value $G_\mathrm{F}^{1/2}$ [5].

As for the interactions of the new vector particles, at present we confine ourselves only to a few remarks. There exist V_iV_jV-, V_iV_jVV-type couplings with the gauge bosons $V = \gamma, W^\pm, Z$, $\overline{f}_1 f_2 V_i V_j$-type interactions and $V_iV_jV_kV_l$-type matter vector couplings always with an even number of V_i. Depending on the mass hierarchy, one or more V_i may be stable. Relatively large are the V_iV_jV couplings.

As an example we consider the Z–B_f–B_2 coupling,

$$L_I = \frac{g}{2\cos\theta_\mathrm{W}}\cos\phi\cdot Z_\mu \tag{25}$$

$$\times\left[B_{f\nu}(\partial^\mu B_2^\nu - \partial^\nu B_2^\mu) - B_{2\nu}(\partial^\mu B_f^\nu - \partial^\nu B_f^\mu)\right]\,.$$

Direct production of B_fB_2 pairs can be studied in high energy e^+e^- colliders, $e^+e^- \to Z^* \to B_fB_2$. Assume in (24) that g_{e^-} is very small; then the direct $e^+e^- \to B_fB_2$ can be neglected. From (25) we have for the total cross section

$$\begin{aligned}&\sigma(e^+e^- \to Z^* \to B_fB_2) \qquad (26)\\ &= \frac{g^4\cos^2\phi}{3\cdot 4096\cos^4\theta_\mathrm{W}}\,\frac{1+(4\sin^2\theta_\mathrm{W}-1)^2}{m^2_{B_2}m^2_{B_f}s^2(s-M_Z^2)^2}\\ &\quad\times\left(s-(m_{B_2}+m_{B_f})^2\right)^{3/2}\cdot\left(s-(m_{B_f}-m_{B_2})^2\right)^{3/2}\\ &\quad\times\left(2s(m^2_{B_2}+m^2_{B_f})+m^4_{B_f}+m^4_{B_2}+10m^2_{B_2}m^2_{B_f}\right)\,.\end{aligned}$$

At asymptotic energies σ is proportional to $1/m^2_{B_2} + 1/m^2_{B_f}$. The mass and energy dependences of σ are shown in Figs. 1 and 2. For example at $\sqrt{s}$= 500 GeV and with an integrated luminosity of $10\,\mathrm{fb}^{-1}$ 5700, 1900, 530 B_fB_2 pairs are expected for $m_{B_f} = m_{B_2}$= 100, 150, 200 GeV and $\cos^2\phi = 1/2$. At $\sqrt{s} = 1.5$ TeV a higher mass range can be tested, for $\cos^2\phi = 1/2$, a luminosity of $100\,\mathrm{fb}^{-1}$, we get the large event numbers 62200, 14500, 5900, 1900, 530 for $m_{B_f} = m_{B_2}$= 100, 200, 300, 400, 500 GeV. One can show that the B^+B^- production is a factor of $\cos^2 2\theta_\mathrm{W}$ smaller than (26) at equal masses and $\cos^2\phi = 1$.

The scale of the model can be estimated most easily applying perturbative unitarity by taking λ_3k_3 negligible in (13); then λ_1 is proportional to $m_+^2G_\mathrm{F}$. Consider $B^\pm B^\pm \to B^\pm B^\pm$ scattering with longitudinally polarized particles and calculate the dominant contact graph contribution [8] to the $J = 0$ partial-wave amplitude. Requiring partial-wave unitarity, $|\mathrm{Re}a_0| \le 1/2$, one gets the maximum possible energy for $2m_+ < \Lambda$. In this way we have that Λ is less than 2–2.5 TeV. This is in agreement with the result of [8]. The bound is similar also for the $B^\pm B^\pm \to B_2B_2$

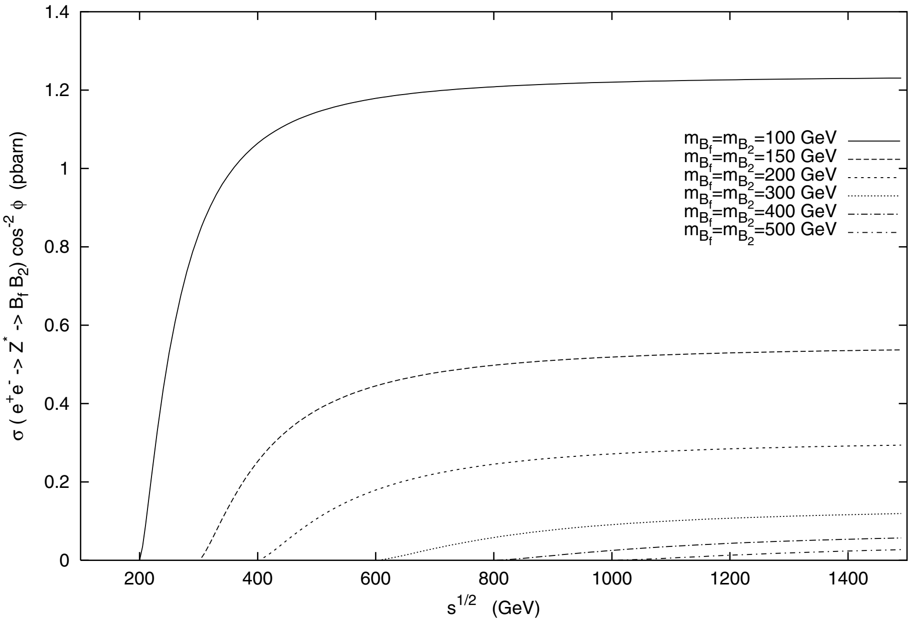

Fig. 2. $\cos^{-2}\phi\,\sigma\left(e^+e^- \to B_f B_2\right)$ versus $\sqrt{s}$ at various $m_{B_f} = m_{B_2}$

scattering. In case of vanishing mixing and k_2, a rough interpretation of k_1 with a cutoff free propagator shows that $m_1, \Lambda \leq$ 2–2.6 TeV for $m_1 < \Lambda$.

In conclusion, a low energy dynamical symmetry breaking model of electroweak interactions based on matter vector field condensation is introduced. Mass generation is arranged starting from gauge invariant Lagrangians. New particles are all spin-one states, one charged pair and three neutral particles having many interactions. The parameter space of the model is larger than that of the one in [5]; therefore, we expect that the positive result of the S, T parameter analysis can be maintained. We hope to investigate the model further in a future work.

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